MASSE

The mass of a body is a theoretical notion corresponding to the intuitive and vague idea of «quantity of matter» contained in a body. It manifests itself first through the force of gravitation that is universally exerted between massive bodies. This «gravitational mass» is directly related to the weight of a body and measures the action of gravity on it. Moreover, mass characterizes the resistance of a body to the modification of its motion: it is the coefficient of inertia, or «inertial mass», of the body. In these two senses, mass is additive according to Newtonian mechanics. Einstein showed in 1905 that this property was only approximate: the mass of a body measures its internal energy (Einstein's relation, $E_0 = mc^2$), and any variation in energy is accompanied by a variation in mass.

Mass and Gravity

At the outset, the notion of mass aims to characterize the «quantity of matter» contained in a physical object. This quantity is first revealed to our senses through the weight of the object: the gravitational force exerted by the Earth is evidently greater the more matter the object contains. Common practice thus tends to assimilate mass and weight, measuring the former by the latter. However, a more careful study reveals that the weight of an object (the gravitational force exerted on it) is not constant on the Earth's surface and varies with latitude and altitude (0.2% more at the equator, and 0.15% less at the summit of Mont Blanc than in Paris). Mass, on the other hand, to characterize the quantity of matter of the object in question as such, must be intrinsic to it and not depend on external conditions. Now, if the weight of an object varies from place to place, it is observed that the ratio of the weights of two given objects is independent of the location and other external conditions. Therefore, one is led to define the ratio of the masses m_1 and m_2 of two objects as equal to the constant ratio of their weights P_1 and P_2: $m_2/m_1 = P_2/P_1$. It then suffices to arbitrarily define a unit mass to measure any mass by comparing the corresponding weights. The conventional unit mass is today the «international kilogram», defined by a platinum-iridium standard kept at the International Bureau of Weights and Measures in Sèvres. This mass corresponds approximately to that of one liter of water; this was the initial definition of the kilogram adopted by the Convention in 1793, but it is insufficiently precise for modern metrology. It is likely that a new definition, based on the masses of atomic objects, will be given to the kilogram in the future, as has been the case for the meter and the second. The notion of mass thus defined is additive. This essential property conforms to the intuitive idea of quantity of matter: the mass (quantity of matter) of a system composed of two objects is the sum of the masses (quantities of matter) of each object. It is moreover this additivity that allows the usual procedure of measuring a mass on a balance by comparison with the cumulative mass of a chosen set of standard weights. Thus, the weight P of an object is the product of its mass m, an intrinsic characteristic of the object, by a quantity g that describes the gravitational field at each point: $P = mg$. It is the variation of the gravitational field according to location that explains the variations in weight. On the Moon, gravity is six times less than on Earth; the televised leaps of astronauts illustrated this reduction in weight, but without change in mass (losing weight is therefore not so difficult; it is to lose mass that one must follow a diet). The gravitational field itself, and this is again a major contribution from Newton, is generated by the masses of bodies other than the one on which it acts. Thus, in the simplest case, two point masses m_1 and m_2 attract each other with a force f:

$$f = Gm_1m_2/d^2,$$

where d is the distance separating them, the famous «inverse square» law, and G a universal constant: Newton's constant, or the gravitational constant. This is the fundamental law of Newton's «universal attraction». It can be understood by considering that the mass m_1 generates a gravitational field that acts on the mass m_2, and vice versa. The similarity of Newton's law with Coulomb's law, which describes the forces between electric charges, leads to thinking of masses as «gravitational charges», in the general sense of the notion of charge in today's physics: a quantity that characterizes both a certain force field generated by a body (active role) and the response of this body to similar force fields generated by others (passive role).

Mass and Inertia

There is another characterization of the mass of a body from its dynamic behavior. Everyday experience indicates it as well: the greater the mass (quantity of matter) of an object, the more difficult it is to set it in motion or to stop it, in other words, to modify its state of motion. The principle of inertia, sketched by Galileo, then by Descartes, and stated by Newton, indicates that a body on which no force acts continues in uniform motion (rectilinear, at constant speed). A given force F will modify the speed in magnitude and/or direction, that is, produce an acceleration γ of the body that is greater the smaller the mass of the body. The fundamental law of dynamics, again due to Newton, is written: $\gamma = F/m$. It perfectly translates the role of mass as an inertial coefficient, characterizing the body's resistance to the change in its state of motion. In this new sense, mass remains an additive property, as can be easily deduced from the additivity of forces. This conception of mass is much more general than the previous one; instead of presenting itself as a quantity linked to a particular physical phenomenon, gravity, it is defined by the response of a body to any force whatsoever, which can be as much electrical, magnetic, etc., as gravitational. One can therefore legitimately distinguish a priori for each body two masses, the inertial mass m_i that intervenes in the (general) law of Newton, $F = m_i\gamma$, and the gravitational mass m_p that characterizes the gravitational force (or weight), $P = m_pg$, acting on the body in a gravitational field g. Observations and experiments confirm that, for all bodies without exception, the ratio m_p/m_i is the same, which allows, with a natural choice of units, to identify m_p and m_i, thus giving a common value (the mass) to the inertial mass and the gravitational mass. The principle of these verifications is simple: if the force acting on the body is gravity ($F = P$), Newton's equation is written: $\gamma = P/m_i = (m_p/m_i)g$. The acceleration, therefore the speed and the trajectory of the body in a given gravitational field, thus depends on the ratio m_p/m_i. Now, it is observed that all bodies have the same motion in a given gravitational field. The fall of bodies on the Earth's surface offers the elementary example: a 10-gram pebble and a 10-ton rock fall with the same acceleration g, «the acceleration due to gravity» ; if the force exerted on the second is a million times stronger than on the first, its inertia is also a million times greater (it is a million times more difficult to accelerate), and there is exact compensation. Similarly, the oscillation period of a pendulum (of given length) is independent of its mass, as is the revolution period of a satellite (of given orbit). Very precise measurements, first carried out by the Hungarian physicist Loránd Eötvös (1890) and, more recently, by the American R. H. Dicke around 1960, have confirmed the universality of the ratio m_p/m_i for a very large number of bodies, with a precision that today reaches 10^{-11}. This concordance between two quantities of very different natures, a gravitational charge and an inertial coefficient, has no natural explanation in the framework of classical physics. Its essential consequence, the identity of the motions of different bodies under the action of a gravitational field, served as a starting point for Einstein to construct his general relativity, in geometric form: it is indeed the independence of the gravitational motion of a body with respect to its own mass that allows considering this motion as due solely to the structure of space-time (cf. SPACE-TIME).

In Einstein's theory, the inertial mass and the (passive) gravitational mass, identified a priori, disappear in a way: the equations of motion determine trajectories in curved space-time by virtue of purely geometric criteria—these are the «geodesics» of space-time. It remains that the very structure of this curved space-time is determined by the (active) gravitational masses of physical bodies.

Mass and Energy

At the beginning of the century, Einstein showed the necessity of modifying the conceptions of Newtonian mechanics by replacing the classical notions of space and time with more elaborate and more intertwined notions. It is known that a major aspect of these new notions is the existence for all material bodies of a limit speed c, which coincides with the speed of light. This implies a modification of Newtonian dynamics: a constant force can no longer correspond to a constant acceleration that would lead, over sufficient time, to a speed greater than the limit c. It is therefore necessary that the inertia of a body increases with its speed, and even increases indefinitely when this speed approaches the limit c, so that it becomes increasingly difficult to accelerate the body and that its speed can only tend asymptotically toward c. In fact, the detailed study of the theory leads to defining an inertia coefficient depending on the speed:

$$I(v) = m/\sqrt{1 - v^2/c^2},$$

which indeed presents these characteristics, m being the mass of the body. For speeds v small compared to c, which are those of common experience and Newtonian physics, one can neglect the term v^2/c^2 compared to unity and write $I \simeq m$, thus recovering the mass as the inertial coefficient. Let us insist on the fact that the mass m remains a constant characteristic of the body, of its quantity of matter. It is incorrect and misleading to include the variation with speed in the notion of mass as is sometimes done: it is better to define the (variable) inertia $I(v)$ and relate it to the (constant) mass m. By exerting a force on a body, one increases, along with its inertia, its energy by communicating kinetic energy to it. Einsteinian relativity reveals a simple relation between the total energy of a body E and its inertia I; it is the fundamental relation $E = Ic^2$ (where c is always the limit speed). Using the expression of inertia as a function of speed, one can write this relation in the form:

$$E = (1/\sqrt{1 - v^2/c^2} - 1)mc^2 + mc^2.$$

The first term, depending on the speed and canceling for $v = 0$, is simply the kinetic energy E_{cin} of the body; in fact, for low speeds v, this term reduces to the classical Newtonian expression $E_{cin} \simeq mv^2/2$. The second term, $E_0 = mc^2$, is the energy of the body at rest ($v = 0$); it is therefore its internal energy, also called mass energy. The identification that thus appears between mass and internal energy (up to the coefficient c^2) is one of the most famous and striking consequences of Einstein's relativity. This is what is called, perhaps a bit hastily, «the equivalence of mass and energy.» It implies a profound conceptual modification of the very notion of mass and, in particular, the loss of its additivity property. Indeed, consider several bodies that unite into another more stable one, for example, the combustion reaction of carbon atoms and oxygen atoms giving birth to carbon dioxide molecules, or the cycle of thermonuclear fusion reactions leading from four hydrogen nuclei to the helium nucleus (source of stellar energy). There is a release of energy (this is the very meaning of stability) and therefore a decrease in the internal energy of the final body compared to the sum of the internal energies of the components. The same goes for the masses, proportional to these internal energies. There is no longer additivity: the final mass shows a «deficit» compared to the sum of the initial masses. The energy release has been at the expense of the total mass, which allows expressing the mass deficit Δm in terms of the loss of internal energy ΔE_0:

$$\Delta m = \Delta E_0/c^2.$$

However, this mass deficit is infinitesimal in daily practice, because the value of the conversion coefficient $1/c^2$ is small. For common mechanical assemblies, the relative mass deficit $\Delta m/m$ hardly exceeds 10^{-15}; for a bound gravitational system, such as the Earth-Moon pair, it is on the order of 10^{-12}; for chemical compounds bound by electrical forces, it barely reaches 10^{-7}. Finally, in the case of nuclear forces, for example, for the aforementioned thermonuclear fusion reaction, it becomes on the order of a thousandth and can approach unity for bonds between fundamental particles. It is seen therefore that the additivity of mass remains valid with excellent approximation in the domain of common sciences and techniques (mechanics, chemistry); it must be seriously questioned only for nuclear and subnuclear phenomena, where the importance of the relative mass deficit is an index of the considerable intensity of the forces involved. We have seen how Einsteinian relativity led to modifying the Newtonian notion of mass. In fact, the most systematic point of view and the most consistent with the principle of relativity consists in defining mass as a «relativistic invariant» linked to the energy and momentum of a body. This means that, if the energy E of a body and its momentum p are variable with its speed and therefore depend on the reference frame used, there exists a combination of these quantities that is invariant and takes the same value in all reference frames. It is precisely the mass m, given by the relation:

$$E^2 - (pc)^2 = (mc^2)^2.$$

At zero speed, in the reference frame where the body is at rest, we have $p = 0$ and we recover $mc^2 = E_0$, giving the internal energy. For comparison, the corresponding

Galilean relation between E, p, and m is written:

$$E = \frac{p^2}{2m} + E_0,$$

where the internal energy E_0 is independent of m. The interest of these considerations is to reveal a new possibility offered by Einstein's theory compared to Newton's theory, namely the possibility of bodies of zero mass, $m = 0$. For these bodies, we have $E = pc$; one can see, from the general expression:

$$E = mc^2/\sqrt{1 - v^2/c^2},$$

that we must always have $v = c$. In other words, such bodies always move at the limit speed c, which is therefore necessarily invariant (independent of the reference frame; this is one of the bases of Einsteinian relativity): increasing or decreasing their energy does not change their speed. There is obviously no reference frame where these bodies are at rest. These bodies are very far from the objects of common experience. At present, known particles of zero mass include the photon, or light quantum (this is why the limit speed c is that of light), and perhaps some of the neutrinos.

Mass and Interactions

We know that matter is composed of atoms, themselves composed of electrons and a nucleus consisting of more fundamental particles: quarks and gluons, which form nucleons, grouped in the nucleus. The mass of any body is therefore provided by that of its constituents, taking into account the mass deficit introduced above. Great interest thus attaches to understanding the masses of particles. Now, in the quantum physics of particles and fundamental interactions, the notion of mass presents several most interesting aspects. Let us first recall that an interaction between two particles is transmitted by the exchange between them of mediating particles, quanta of the interaction field, such as the photon, vector of the electromagnetic interaction between charged particles (cf. QUANTUM PHYSICS). These intermediate quanta are said to be «virtual» ; their ephemeral existence, between their emission and absorption, cannot exceed, under penalty of violating the law of energy conservation, a duration Δt linked, by Heisenberg's inequality, to the energy dispersion $\Delta E \geqslant mc^2$ necessary to ensure their appearance:

$$\Delta t \leqslant \hbar/mc^2,$$

where $\hbar$ is Planck's constant. A physical action cannot propagate at a speed greater than the limit speed c, so the range of the interaction mediated by the quantum cannot act at a distance greater than $c\Delta t$. Consequently, the range a of an interaction is linked to the mass m of the quantum that transmits it by:

$$a \simeq \hbar/mc.$$

It is this reasoning that allowed Hideki Yukawa, in 1935, to predict the mass of the π mesons, presumed responsible for nuclear interactions. The infinite range of electromagnetic forces is explained by the same formula, by the zero mass of the photon (cf. nuclear PHYSICS). It would be desirable, moreover, that the theory allow understanding and calculating the values of the masses of particles.
However, despite the great progress in the theory of fundamental interactions, the masses of particles pose serious problems there. Some facts are clear: the masses

of particles are greater the more intense the interactions that govern them. Thus, leptons (electrons, muons, neutrinos) which are not subject to strong nuclear interactions are clearly lighter than hadrons (nucleons, mesons, etc.) which are subject to them. This is explained: if the mass of a particle is its internal energy (up to $1/c^2$), it must include the energy of which the various fields generated by the particle are the seat, contributions all the stronger as these fields correspond to more intense interactions. Consider, for example, the case of electromagnetism. Suppose a hypothetical electron devoid of electric charge (it is said to be «bare») has a mass m_0. For a real, charged electron, the electromagnetic field surrounding it carries an energy and therefore a contribution δm to the total mass m, which is thus the sum of these two terms:

$$m = m_0 + \delta m.$$

Only the total mass m is observable, although the theory is initially expressed in terms of the bare mass m_0. It is therefore necessary to «renormalize» the theory, to express it in terms of the (renormalized) mass m. Now, the calculation, in quantum electrodynamics, provides an infinite value for the interactive «correction» δm, which obviously implies an equally infinite value for the bare mass m_0 (since m is finite). Despite this strange situation, the renormalization of the theory can be carried out coherently and the infinities thus appeared can be eliminated. This procedure leads to admirably precise results, but its mathematical foundations remain open to questioning. Moreover, in the case of strong interactions (quantum chromodynamics), it is not known at present how to carry this procedure to completion (cf. ELEMENTARY PARTICLES). The fundamental problem remains to understand the mass scales of particles within each family itself. Thus, no sketch of an explanation yet exists to understand the masses of charged leptons—electron, muons, tauon—which are in the respective ratios 1:207:34. Regarding the masses of their neutrinos, it is not known if they are rigorously zero, or if, as some recent indications suggest, muonic and tauonic neutrinos have weak but non-zero masses. Finally, the order of magnitude itself of these masses, for example that of the electron $m_e = 0.9 \times 10^{-30}$ kg, remains mysterious (its mass energy is generally given in electronvolts, namely $E_e = m_e c^2 = 0.51$ MeV). The same questions arise for the quark family. These questions are of capital interest not only for particle physics but also for cosmology. Various problems regarding the initial evolution of the Universe or its present structure (for example, its open or closed character) depend on the masses of these constituents—known or not. Thus, the existence of a non-zero neutrino mass could qualitatively modify the total energy content of the Universe, as could the possible presence of hypothetical heavy particles (monopoles, axions?). It is possible that solving the enigmas posed by the masses of the fundamental constituents of matter requires the formulation of radically new ideas.

JEAN-MARC LÉVY-LEBLOND

M. FANTIN, « Le Kilogramme américain rentre dans le rang », in *La Recherche*, n° 178, p.856, juin 1986 / R. FEYNMAN, *The Charactere of Physical Law*, MIT Press, 1964 / C. KITTEL, W. D. KNIGHT & M. A. RUDERMAN, *Mechanics*, Berkeley Physics Course, 1965 / L. LANDAU & E. LIFCHITZ, *Mechanics*, Pergamon Press, 1960 / M. LE BELLAC, *Introduction à la mécanique*, Belin, Paris, 1985 / J. ROSMORDUC, *Une histoire de la physique et de la chimie*, Seuil, 1986 / E. TAYLOR

& J. Wheeler, *Spacetime physics*, San Francisco : W. H. Freeman, 1966 / M.-A. Tonnelat, *Histoire du principe de relativité*, Flammarion, Paris, 1971.